Investigation of $B_{u,d} \to (\pi,\, K)\pi$ decays within unparticle physics

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Abstract

We investigate the implication of unparticle physics on the $B_{u,d} \to (\pi,\, K)\pi$ decays under the constraints of the $B_{d,s} - \bar{B}_{d,s}$ mixing. We found that not only the unparticle parameters that belong to the flavor changing neutral current (FCNC) processes but also scaling dimension $d_{\mathcal{U}}$ could be constrained by the $B_{d,s} - \bar{B}_{d,s}$ mixing phenomenology. Employing the minimum χ^2 analysis to the $B_{u,d} \to (\pi,\, K)\pi$ decays with the constraints of $B_{d,s}$ mixing, we find that the puzzle of large branching ratio for $B_d \to \pi^0\pi^0$ and the discrepancy between the standard model estimation and data for the direct CP asymmetry of $B^+ \to K^+\pi^0$ and $B_d \to \pi^+\pi^-$ can be resolved well. However, the mixing induced CP asymmetry of $B_d \to K_S\pi^0$ could not be well accommodated by the unparticle contributions.

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I. INTRODUCTION

Recently some incomprehensible phenomena at B factories have been explored, especially $B_{u,d} \to (\pi,\, K)\pi$ decays. Firstly, the observations on the large branching ratio (BR) for $B_d \to \pi^0\pi^0$ decay with the world average $\mathcal{B}(B_d \to \pi^0\pi^0) = (1.31 \pm 0.21) \times 10^{-6}$ and the direct CP asymmetry for $B_d \to \pi^+\pi^-$ with $\mathcal{A}_{CP}(B_d \to \pi^+\pi^-) = 0.38 \pm 0.07$ [1] are inconsistent with the theoretical estimations of around 0.5×10^{-6} and $10 - 20\%$, respectively. Secondly, a disagreement in the CP asymmetries (CPAs) for $B_d \to K^+\pi^-$ and $B^+ \to K^+\pi^0$ has been observed to be -0.097 ± 0.012 and 0.050 ± 0.025 [1], respectively, while the naive estimation is $\Delta_{CP} \equiv \mathcal{A}_{CP}(B^+ \to K^+\pi^0) - \mathcal{A}_{CP}(B_d \to K^+\pi^-) \sim 0$. Although many theoretical calculations based on QCDF [2], PQCD [3] and SCET [4] have been tried to produce the consistencies with data in the framework of standard model (SM), however, the results have not been conclusive yet [5]. For instance, the recent PQCD result for the Δ_{CP} is 0.08 ± 0.09, which is actually consistent with the data. However, the PQCD prediction $\mathcal{A}_{CP}(B^+ \to K^+\pi^0)_{PQCD} = -0.01^{+0.03}_{-0.05}$ still has 1.4σ difference from the current experimental data [6]. In addition, the difference between $(\sin 2\beta)_{K_S\pi^0}$ and $(\sin 2\beta)_{J/\Psi K_S}$ in the mixing-induced CPA from the PQCD prediction is 0.065 ± 0.04, which shows about 2σ off the data -0.30 ± 0.19. Hence, the inconsistencies between data and theoretical predictions provide a strong indication to investigate the new physics beyond SM.

There introduced many extensions of the SM, and enormous studies have been done on searching some specific models beyond SM, e.g. on supersymmetric model [7], extra-dimension model [8], left-right symmetric model [9] and flavor-changing Z' model [10]. Although new physics effects will be introduced, however, in phenomenological sense we just bring more particles and their related interactions to our system. Recently, Georgi proposed completely different stuff and suggested that an invisible sector, dictated by the scale invariance and coupled weakly to the particles of the SM, may exist in our universe [11, 12]. Unlike the concept of particles in the SM or its normal extensions where the particles own the definite mass, the scale invariant stuff cannot have a definite mass unless it is zero. Therefore, if the peculiar stuff exists, it should be made of *unparticles* [11]. Furthermore, in terms of the two-point function with the scale invariance, it is found that the unparticle with the scaling dimension $d_{\mathcal{U}}$ behaves like a non-integral number $d_{\mathcal{U}}$ of invisible particles [11]. Based on Georgi's proposal, the phenomenology of unparticle physics has been extensively

studied in Refs. [11, 12, 13, 14, 15, 16]. For illustration, some examples such as $t \to u + \mathcal{U}$ and $e^+e^- \to \mu^+\mu^-$ have been introduced to display the unparticle properties. In addition, it is also suggested that the unparticle production in high energy colliders might be detected by searching for the missing energy and momentum distributions [11, 12, 13]. Nevertheless, we have to point out that flavor factories with high luminosities, such as SuperKEKB [17], SuperB [18] and LHCb [19] etc, should also provide good environments to search for the unparticle effects in indirect way.

Besides the weird property of non-integral number of unparticles, the most astonished effect is that an unparticle could carry a peculiar CP conserving phase associated with its propagator in the time-like region [12, 13]. It has been pointed out that the unparticle phase plays a role like a strong phase and has an important impact on direct CP violation (CPV) [14]. In this paper, we will make detailed analysis to examine whether the puzzles in $B_{u,d} \to (\pi, K)\pi$ decays with the $B_{d,s} - \bar{B}_{d,s}$ mixing constraints could be resolved when the invisible unparticle stuff is introduced to the SM.

In order to study the flavor physics associated with scale invariant stuff, we follow the scheme proposed in Ref. [11]. For the system with the scale invariance, there exist so-called Banks-Zaks ($\mathcal{BZ}$) fields that have a nontrivial infrared fixed point at a very high energy scale [20]. Subsequently, with the dimensional transmutation at the $\Lambda_{\mathcal{U}}$ scale, the $\mathcal{BZ}$ operators composed of $\mathcal{BZ}$ fields will match onto unparticle operators. We consider only vector unparticle operator in the following analysis. Then, the effective interactions for unparticle stuff and the particles of the SM are adopted to be

$$\frac{C_L^{q'q}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{q}'\gamma_\mu(1-\gamma_5)q\mathcal{O}_{\mathcal{U}}^\mu + \frac{C_R^{q'q}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{q}'\gamma_\mu(1+\gamma_5)q\mathcal{O}_{\mathcal{U}}^\mu , \tag{1}$$

where $C_{L,R}^{q'q}$ are effective coefficient functions and $\mathcal{O}_{\mathcal{U}}^\mu$ denotes the spin-1 unparticle operator with scaling dimension $d_{\mathcal{U}}$ and is assumed to be hermitian and transverse $\partial_\mu \mathcal{O}_{\mathcal{U}}^\mu = 0$. Since so far the theory for $\mathcal{BZ}$ fields and their interactions with SM particles is uncertain, here $C_{L,R}^{q'q}$ are regarded as free parameters. With scale invariance, the propagator of vector unparticle can be obtained by [12, 13]

$$\begin{aligned} &\int d^4x e^{ip\cdot x} \langle 0|T\left(O_{\mathcal{U}}^\mu(x) O_{\mathcal{U}}^\nu(0)\right)|0\rangle \\ &= i\Delta_{\mathcal{U}}(p^2)\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}\right) e^{-i\phi_{\mathcal{U}}} , \end{aligned} \tag{2}$$

with $\phi_{\mathcal{U}} = (d_{\mathcal{U}} - 2)\pi$ and

$$\begin{aligned}
\Delta_{\mathcal{U}}(p^2) &= \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)} \frac{1}{(p^2 + i\epsilon)^{2-d_{\mathcal{U}}}} \,, \\
A_{d_{\mathcal{U}}} &= \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})} \,,
\end{aligned} \tag{3}$$

where $\phi_{\mathcal{U}}$ could be regarded as a CP conserving phase [12, 13, 14]. We note that when unparticle stuff is realized in the framework of conformal field theories, the propagators for vector and tensor unparticles should be modified [21]. Although conformal invariance typically implies scale invariance, however, in principle it is not necessary. Unparticle stuff with scalar invariance in 2D spacetime has been investigated in Ref. [22]. Hence, in this work, we still concentrate on the stuff built out of only scale invariance. For simplicity, we set the unknown scale factor $\Lambda_{\mathcal{U}}$ to be 1 TeV throughout the analysis.

II. CONSTRAINTS OF $B_{d,s} - \bar{B}_{d,s}$ MIXING

Since the tree level FCNC processes are allowed in the unparticle physics, it is expected that the $B_{d,s} - \bar{B}_{d,s}$ mixings offer strong constraints on the unparticle parameters of C_L^{db}, C_R^{db} and C_L^{sb}, C_R^{sb}. Moreover, it will be shown that the scaling dimension $d_{\mathcal{U}}$ also can be constrained by $B_{d,s} - \bar{B}_{d,s}$ mixings.

First of all, we separate the matrix element of $\Delta B = 2$ transition for the $B_q^0 - \bar{B}_q^0$ mixing ($q = d, s$), which is denoted by M_{12}^q, into the SM and unparticle contribution as follows.

$$M_{12}^q = M_{12}^{q,SM} + M_{12}^{q,NP} = |M_{12}^{q,SM}| e^{i\phi_q^{SM}} + |M_{12}^{q,NP}| e^{i\phi_q^{NP}} \,. \tag{4}$$

where the ϕ_q^{SM} and ϕ_q^{NP} represent the phases of mixing amplitudes. For the second term, we use the superscript *'NP'* in order to represent general new physics (NP) contribution. Later on, we regard this $M_{12}^{q,NP}$ as the unparticle mixing amplitude.

As is well known, the magnitude of total mixing amplitude $|M_{12}^q|$ is given by the $B_q^0 - \bar{B}_q^0$ oscillating frequency as follows

$$\Delta M_q = 2|M_{12}^q| \,. \tag{5}$$

And the mixing phase $\phi_q \equiv \arg M_{12}^q$ can be obtained from the mixing induced CP asymmetry of $b \to c\bar{c}s$ processes. We summarize current experimental data in Table I.

TABLE I: Experimental values for the $B_{d,s} - \bar{B}_{d,s}$ mixings. Even though D0 collaboration recently have measured ϕ_s [23], the data is not used in our analysis because of huge error of it.

observables	values	note
ΔM_d	$(\ 0.507 \pm 0.004 \ ) \ \mathrm{ps}^{-1}$	HFAG [1]
ΔM_s	$(\ 17.77 \pm 0.12 \ ) \ \mathrm{ps}^{-1}$	CDF [24]
ϕ_d	$43^\circ \pm 2^\circ$	HFAG [1]

The SM mixing amplitude reads

$$M_{12}^{q,SM} = \frac{G_F^2 m_W^2}{12\pi^2} m_{B_q} f_{B_q}^2 \hat{B}_{B_q} (V_{tq}^* V_{tb})^2 \hat{\eta}^B S_0(x_t) \,, \tag{6}$$

where $\hat{\eta}^B = 0.552$ is short distance QCD correction term [25], and $S_0(x_t) = 2.35 \pm 0.06$ is an Inami-Lim function for the t-quark exchange in the loop diagram [26]. The quantities of f_{B_q} and B_{B_q} are non-perturbative parameters which can be obtained from the lattice calculations. We follow the procedure given in Ref. [27] for dealing with these non-perturbative parameters. The procedure mainly employs the result of lattice calculations in two different ways. The one is to use the result of JLQCD collaboration [28], and the other is to combine the results of JLQCD and HPQCD [29] collaborations. We note that one can obtain the SM mixing phases from Eq.(6) as follows.

$$\phi_d^{SM} = 2\beta, \qquad \phi_s^{SM} = -2\lambda^2\eta \,, \tag{7}$$

where β is an angle of CKM unitarity triangle, λ and η are from the Wolfenstein parametrization [30]. We use the result of UTfit [31] from the tree level processes for the β, R_t and $\bar{\eta}$, where $R_t \equiv \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}$ and $(\bar{\rho}, \bar{\eta})$ is the apex of the CKM unitary triangle. For the $|V_{cb}|$, we adopt the result of global fit to moment of inclusive distributions in $B \to X_c l \nu_l$, which is performed in the framework of heavy quark expansions with kinetic scheme [32]. After putting all the SM input parameters into Eq. (6), the SM mixing amplitudes are obtained. And using the experimental data shown in the Table I, the NP mixing amplitudes for the $B_d - \bar{B}_d$ mixing could be gained through the Eq. (4). The numerical values are summarized in Table II.

As for the unparticle contribution to the mixing amplitude, we begin with the effective

TABLE II: Numerical values for the SM $B_{d,s} - \bar{B}_{d,s}$ mixing amplitudes. The values of the NP $B_d - \bar{B}_d$ mixing amplitude are obtained from the experimental data and the Eq. (4). The case (a) denotes JLQCD, while the case (b) denotes (HP+JL)QCD. $M_{12}^{q,NP} = M_{12}^{q,\mathcal{U}}$ should be considered $(q = s, d)$.

SM parameters	values		NP parameters	values	
$2\|M_{12}^{d,SM}\|$	$0.75^{+0.20}_{-0.26}$ ps^{-1}	(a)	$2\|M_{12}^{d,NP}\|$	0.25 ± 0.26 ps^{-1}	(a)
	0.97 ± 0.29 ps^{-1}	(b)		0.46 ± 0.29 ps^{-1}	(b)
ϕ_d^{SM}	$45.2° \pm 5.7°$		ϕ_d^{NP}	$-130° \pm 180°$	(a)
				$-132° \pm 12°$	(b)
$2\|M_{12}^{s,SM}\|$	16.4 ± 2.8 ps^{-1}	(a)	$2\|M_{12}^{s,NP}\|$	-	
	23.8 ± 5.9 ps^{-1}	(b)			
ϕ_s^{SM}	$-2.3° \pm 0.2°$		ϕ_s^{NP}	-	

Hamiltonian for $\Delta B = 2$ processes in unparticle sector such as

$$\begin{aligned}
\mathcal{H}^{q,\mathcal{U}} &= 2 \cdot \frac{1}{4} \cdot \left(\frac{p^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1} \frac{1}{p^2} \frac{A_{\mathcal{U}}}{2\sin d_{\mathcal{U}}\pi} e^{-i\phi_{\mathcal{U}}} \\
&\times \left[-\bar{q}\gamma_\mu \left(C_L^{qb}(1-\gamma_5) + C_R^{qb}(1+\gamma_5)\right) b\, \bar{q}\gamma^\mu \left(C_L^{qb}(1-\gamma_5) + C_R^{qb}(1+\gamma_5)\right) b \right. \\
&+ \left. \frac{1}{p^2}\bar{q}\, \not{p} \left(C_L^{qb}(1-\gamma_5) + C_R^{qb}(1+\gamma_5)\right) b\, \bar{q}\, \not{p} \left(C_L^{qb}(1-\gamma_5) + C_R^{qb}(1+\gamma_5)\right) b\right] . \qquad (8)
\end{aligned}$$

The factor 2 is from the fact that there are s and $t-$channel which give same result, and the factor 1/4 is due to the Wick contraction factor [33]. From this effective Hamiltonian, the transition matrix elements can be shown as

$$M_{12}^{q,\mathcal{U}} = -\frac{\Delta_{\mathcal{U}}(p^2)}{(\Lambda_{\mathcal{U}}^2)^{d_{\mathcal{U}}-1}} e^{-i\phi_{\mathcal{U}}} m_{B_q} f_{B_q}^2 \hat{B}_{B_q} a_{\rm mix}^{q,\mathcal{U}} , \qquad (9)$$

where the $a_{\rm mix}^{q,\mathcal{U}}$ is defined by

$$a_{\rm mix}^{q,\mathcal{U}} \equiv \left[(C_L^{qb})^2 + (C_R^{qb})^2\right] \left[\frac{2}{3} - \frac{5}{12}\frac{m_{B_q}^2}{p^2}\right] + C_L^{qb} C_R^{qb} \left[-\frac{5}{3} + \frac{7}{6}\frac{m_{B_q}^2}{p^2}\right] . \qquad (10)$$

with $p^2 = m_{B_q}^2$. $\Delta_{\mathcal{U}}(p^2)$ is given in Eq. (3). In order to get the constraints on the unparticle parameters from the NP parameter regions shown in the Table II, we first consider the phase of $M_{12}^{q,\mathcal{U}}$. It depends on the scaling dimension $d_{\mathcal{U}}$ through the $e^{-i\phi_{\mathcal{U}}}$ term and the sign of $\sin(d_{\mathcal{U}}\pi)$ in the $\Delta_{\mathcal{U}}(p^2)$. The plot of $\arg(M_{12}^{d,\mathcal{U}})$ versus $d_{\mathcal{U}}$ is shown in Fig 1.

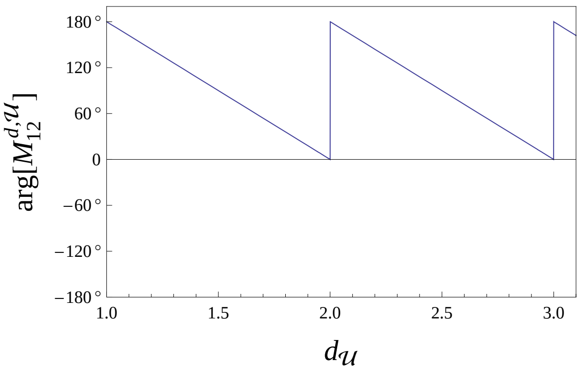

FIG. 1: Plot of $\arg(M_{12}^{d,\mathcal{U}})$ versus $d_{\mathcal{U}}$ within the range of $(-180^\circ, 180^\circ)$.

It is interesting that $\arg(M_{12}^{d,\mathcal{U}})$ takes only positive value as displayed in Fig. 1. Therefore, as we can see from Table II. (HP+JL)QCD case can not give a solution of $d_{\mathcal{U}}$ while all value of $\arg(M_{12}^{d,\mathcal{U}})$ is possible in JLQCD case. So, we take only the JLQCD case. Although we can not determine $d_{\mathcal{U}}$ in the JLQCD case presently, if the SM prediction for the $B_d - \bar{B}_d$ mixing amplitude becomes more precise in future, it can give strong constraint on $d_{\mathcal{U}}$ from the plot of $\arg(M_{12}^{d,\mathcal{U}})$ versus $d_{\mathcal{U}}$. Here, we assume $d_{\mathcal{U}} = 1.5$ for the remaining analysis. Then, the magnitude of unparticle transition matrix element $M_{12}^{s,\mathcal{U}}$ can be obtained through Eq. (4) and the values in Table II for the JLQCD case with the experimental data as $2|M_{12}^{s,\mathcal{U}}| = 7.6 \pm 6.6$ ps^{-1}. Using this value and the value for $2|M_{12}^{d,\mathcal{U}}|$ in Table II, we can see that the mixing parameter $a_{\rm mix}^{q,\mathcal{U}}$ should be strongly suppressed as follows.

$$|a_{\rm mix}^{d,\mathcal{U}}| = (1.1 \pm 1.3) \times 10^{-8}\,, \qquad |a_{\rm mix}^{s,\mathcal{U}}| = (2.6 \pm 2.3) \times 10^{-7}\,. \tag{11}$$

And, the Eq. (10) leads to

$$|C_L^{qb} - C_R^{qb}| = 2\sqrt{a_{\rm mix}^{q,\mathcal{U}}}\,. \tag{12}$$

Therefore, the parameters C_L^{qb} and C_R^{qb} $(q = d, s)$ are strongly correlated under the $B_{d,s} - \bar{B}_{d,s}$ mixing. In the next section, these strong constraints on the unparticle parameters will be used for fitting the parameters in $B_{u,d} \to (\pi,\, K)\pi$ decays

III. $B_{u,d} \to (\pi,\, K)\pi$ DECAYS AND THE UNPARTICLE CONTRIBUTIONS

According to the Eqs. (1) and (2), the effective Hamiltonian for $b \to q\bar{q}'q'$ decays is obtained by

$$\mathcal{H}_{\mathcal{U}} \;=\; -C_{\mathcal{U}}(q^2)\left(C_L^{qb}(\bar{q}b)_{V-A} + C_R^{qb}(\bar{q}b)_{V+A}\right)\left(C_L^{q'q'}(\bar{q}'q')_{V-A} + C_R^{q'q'}(\bar{q}'q')_{V+A}\right)\,, \quad (13)$$

where $q = (d,\, s)$, $q' = (u,\, d,\, s,\, c)$, $(\bar{f}'f)_{V\pm A} = \bar{f}'\gamma_\mu(1 \pm \gamma_5)f$ and

$$C_{\mathcal{U}}(q^2) = \frac{\Delta_{\mathcal{U}}(q^2)}{(\Lambda^2_{\mathcal{U}})^{d_{\mathcal{U}}-1}} e^{-i\phi_{\mathcal{U}}}\,. \quad (14)$$

Based on this effective Hamiltonian, we study the unparticle contributions to the decay amplitudes for $B_{u,d} \to (\pi,\, K)\pi$. It is known that the most uncertain theoretical calculations for two-body exclusive decays are the QCD hadronic transition matrix elements. To deal with the hadronic matrix elements, we adopt recent perturbative QCD (PQCD) calculations for the SM amplitudes and naive factorization (NF) approach for the amplitudes of unparticle contributions. The SM amplitudes for $B_{u,d} \to (\pi,\, K)\pi$ decays can be parameterized in the context of quark diagram approach (QDA) [34] as follows

$$\sqrt{2}A^{SM}(B^+ \to \pi^+\pi^0) \;=\; -Te^{i\gamma} - Ce^{i\gamma} - P_{EW}e^{-i\beta}, \quad (15)$$

$$A^{SM}(B_d \to \pi^+\pi^-) \;=\; -Te^{i\gamma} - Pe^{-i\beta}, \quad (16)$$

$$\sqrt{2}A^{SM}(B_d \to \pi^0\pi^0) \;=\; -Ce^{i\gamma} + Pe^{-i\beta} - P_{EW}e^{-i\beta}, \quad (17)$$

$$A^{SM}(B^+ \to K^0\pi^+) \;=\; P', \quad (18)$$

$$A^{SM}(B_d \to K^+\pi^-) \;=\; -P' - T'e^{i\gamma}, \quad (19)$$

$$\sqrt{2}A^{SM}(B^+ \to K^+\pi^0) \;=\; -P' - T'e^{i\gamma} - C'e^{i\gamma} - P'_{EW}, \quad (20)$$

$$\sqrt{2}A^{SM}(B_d \to K^0\pi^0) \;=\; P' - C'e^{i\gamma} - P'_{EW}\,, \quad (21)$$

where $T^{(\prime)}$ and $C^{(\prime)}$ denote the tree color-allowed and -suppressed amplitudes for $B_{u,d} \to \pi(K)\pi$, respectively, while $P^{(\prime)}(P^{(\prime)}_{EW})$ is gluonic (electroweak) penguin amplitude. All CP-conserving phases are included in these parameters. The phase $\gamma(\beta)$ is the CP violating phase in the SM and from $V_{ub}(V_{td})$. Table III shows the recent PQCD result for the values of each topological parameters [6, 35].

For deriving the unparticle contributions, the definitions for relevant decay constants and

TABLE III: Recent PQCD predictions for the topological parameters of $B_{u,d} \to (\pi,\, K)\pi$ in unit of 10^{-5} GeV . The phases are indicating strong phases of the parameters in radian unit. The predictions include NLO calculation.

Topology	Abs	Arg	Topology	Abs	Arg
P'	$43.6^{+10.8}_{-8.0}$	$2.9^{+0.1}_{-0.2}$	T	$23.2^{+8.0}_{-6.1}$	0.0 ± 0.0
T'	$6.5^{+2.4}_{-1.8}$	0.1 ± 0.0	P	$5.6^{+1.2}_{-0.8}$	$-0.4^{+0.2}_{-0.1}$
P'_{EW}	$5.4^{+1.4}_{-1.0}$	-1.3 ± 0.1	C	$4.3^{+2.1}_{-1.5}$	-1.1 ± 0.0
C'	$1.7^{+0.9}_{-0.6}$	-3.0 ± 0.0	P_{EW}	$0.7^{+0.1}_{-0.1}$	-0.1 ± 0.0

form factors are given by

$$\begin{aligned}
\langle P(p)|\bar{q}\gamma_\mu\gamma_5 u|0\rangle &= i f_P p_\mu\,, \\
\langle P(p)|\bar{q}\gamma_5 u|0\rangle &= i f_P m_P^0\,, \\
\langle P(p)|\bar{q}\gamma_\mu b|\bar{B}(p_B)\rangle &= \left[(p_B+p)_\mu - \frac{m_B^2}{q^2}\, q_\mu\right] F_1^{BP}(q^2) + \frac{m_B^2}{q^2}\, q_\mu F_0^{BP}(q^2)\,, \qquad (22)
\end{aligned}$$

with $P = (\pi,\, K)$, $q = p_B - p$ and $m_P^0 = m_P^2/(m_q + m_u)$. Here, due to $m_P \ll m_B$, we have neglected the m_P^2 effects in $B \to P$ transition matrix element. Subsequently, by considering various flavor diagrams in which the typical diagrams mediated by unparticle are illustrated in Fig. 2, the unparticle amplitudes for $B_{u,d} \to (\pi,\, K)\pi$ decays within NF approach are

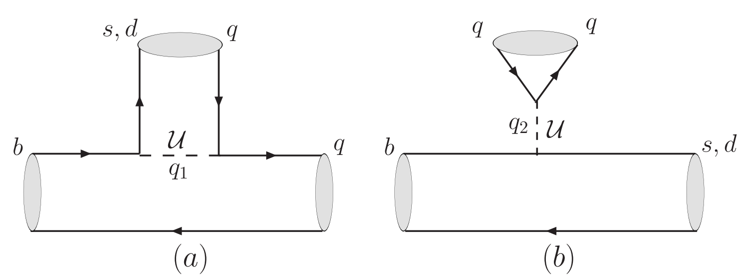

FIG. 2: Typical flavor diagrams for $B \to (\pi, K)\pi$ decays mediated by unparticle with q=u and d, where $q_{1,2}$ are the momenta of unparticle.

obtained to be

$$A^{\mathcal{U}}(B \to \pi^i\pi^j) = C_{\mathcal{U}}(q_1^2) f_\pi m_B^2 F_0^{B\pi}(m_\pi^2) a_{\rm dec}^{\mathcal{U},\pi^i\pi^j}\,, \qquad (23)$$

$$A^{\mathcal{U}}(B \to K^i\pi^j) = C_{\mathcal{U}}(q_1^2) f_K m_B^2 F_0^{B\pi}(m_K^2) a_{\rm dec}^{\mathcal{U},K^i\pi^j}\,, \qquad (24)$$

TABLE IV: The definition of coefficients for the unparticle amplitudes in $B_{u,d} \to (\pi,\, K)\pi$ decays within NF approach.

decay mode	$a^{\mathcal{U}}_{\rm dec}$
$\pi^+\pi^-$	$-\frac{1}{N_c}\Big(\big(C^{db}_L C^{uu}_L - C^{db}_R C^{uu}_R\big) + 2r^{\pi}_1 \big(C^{db}_L C^{uu}_R - C^{db}_R C^{uu}_L\big) \Big)$
$\pi^+\pi^0$	$-\frac{1}{\sqrt{2}N_c}\Big(\big(C^{db}_L (C^{uu}_L - C^{dd}_L) - C^{db}_R (C^{uu}_R - C^{dd}_R)\big)$ $+2r^{\pi}_2 \big(C^{db}_L (C^{uu}_R - C^{dd}_R) - C^{db}_R (C^{uu}_L - C^{dd}_L)\big) \Big)$ $-\frac{C_{\mathcal{U}}(q_2^2)}{\sqrt{2}C_{\mathcal{U}}(q_1^2)} \big(C^{db}_L + C^{db}_R\big)\big(C^{uu}_L - C^{dd}_L - C^{uu}_R + C^{dd}_R\big)$
$\pi^0\pi^0$	$\frac{1}{\sqrt{2}N_c}\Big(\big(C^{db}_L C^{dd}_L - C^{db}_R C^{dd}_R\big) + 2r^{\pi}_2 \big(C^{db}_L C^{dd}_R - C^{db}_R C^{dd}_L\big) \Big)$ $-\frac{C_{\mathcal{U}}(q_2^2)}{\sqrt{2}C_{\mathcal{U}}(q_1^2)} \big(C^{db}_L + C^{db}_R\big)\big(C^{uu}_L - C^{dd}_L - C^{uu}_R + C^{dd}_R\big)$
$K^0\pi^-$	$\frac{1}{N_c}\Big(\big(C^{sb}_L C^{dd}_L - C^{sb}_R C^{dd}_R\big) + 2r^{K}_1 \big(C^{sb}_L C^{dd}_R - C^{sb}_R C^{dd}_L\big) \Big)$
$K^+\pi^-$	$-\frac{1}{N_c}\Big(\big(C^{sb}_L C^{uu}_L - C^{sb}_R C^{uu}_R\big) + 2r^{K}_1 \big(C^{sb}_L C^{uu}_R - C^{sb}_R C^{uu}_L\big) \Big)$
$K^+\pi^0$	$-\frac{1}{\sqrt{2}N_c}\Big(\big(C^{sb}_L C^{uu}_L - C^{sb}_R C^{uu}_R\big) + 2r^{K}_1 \big(C^{sb}_L C^{uu}_R - C^{sb}_R C^{uu}_L\big) \Big)$ $-\frac{C_{\mathcal{U}}(q_2^2)}{\sqrt{2}C_{\mathcal{U}}(q_1^2)} \frac{f_\pi}{f_K} \frac{F_0^{BK}(m_\pi^2)}{F_0^{B\pi}(m_K^2)} \big(C^{sb}_L + C^{sb}_R\big)\big(C^{uu}_L - C^{dd}_L - C^{uu}_R + C^{dd}_R\big)$
$K^0\pi^0$	$\frac{1}{\sqrt{2}N_c}\Big(\big(C^{sb}_L C^{dd}_L - C^{sb}_R C^{dd}_R\big) + 2r^{K}_2 \big(C^{sb}_L C^{dd}_R - C^{sb}_R C^{dd}_L\big) \Big)$ $-\frac{C_{\mathcal{U}}(q_2^2)}{\sqrt{2}C_{\mathcal{U}}(q_1^2)} \frac{f_\pi}{f_K} \frac{F_0^{BK}(m_\pi^2)}{F_0^{B\pi}(m_K^2)} \big(C^{sb}_L + C^{sb}_R\big)\big(C^{uu}_L - C^{dd}_L - C^{uu}_R + C^{dd}_R\big)$

where the coefficients $a^{\mathcal{U}}_{\rm dec}$s are defined in Table IV, $N_c = 3$ is the number of colors, $q_2^2 = m_\pi^2$, and the chiral enhanced factor $r^{\pi}_{(1,2)}$ and $r^{K}_{(1,2)}$ are defined by

$$
\begin{aligned}
r^{\pi}_1 &= \frac{m_\pi^2}{m_b(m_u+m_d)}\,, \qquad r^{\pi}_2 = \frac{m_\pi^2}{m_b(m_d+m_d)}\,, \\
r^{K}_1 &= \frac{m_K^2}{m_b(m_u+m_s)}\,, \qquad r^{K}_2 = \frac{m_K^2}{m_b(m_d+m_s)}\,.
\end{aligned} \tag{25}
$$

We note that since q_1 in Fig. 2(a) involves different mesons, the estimation of q_1^2 should have ambiguity. To understand the typical value of q_1^2, we write the $q_1 = p_B - k_2 - k_3$ with $k_{2,3}$ being the momenta of valence quarks inside the light mesons. In terms of momentum fraction of valence quark and light-cone coordinates and by neglecting the transverse momentum, one can get $k_2 = (0, m_B x_2/\sqrt{2}, \vec{0}_\perp)$ and $k_3 = (m_B x_3/\sqrt{2}, 0, \vec{0}_\perp)$. As a result, we have $q_1^2 = m_B^2(1-x_2)(1-x_3)$. According to the behavior of leading twist wave function of light meson, $\Phi^{\rm tw-2} \propto x(1-x)$ which is calculated by QCD sum rules [37], it is known that the maxima of $x_{2,3}$ occur at $x_2 = x_3 \sim 1/2$. Therefore, for numerical estimations, the momentum transfer could be roughly taken as $q_1^2 \approx m_B^2/4$ with $m_B = 5.28$ GeV.

We discard irrelevant factor i in the NF and match the sign with the QDA parametrization; then, the total amplitude is

$$A(B \to f) = \kappa_f A^{SM}(B \to f) + A^{\mathcal{U}}(B \to f)\,. \tag{26}$$

Here, the κ_f is the ratio of phase space factor coming from the difference of notation of decay amplitude between NF and PQCD group and is defined by

$$\kappa_f \equiv \sqrt{\left(\frac{G_F^2 m_b^3}{128\pi}\right) \Big/ \left(\frac{p_f}{8\pi m_B^2}\right)} = 1.15 \times 10^{-4}\,. \tag{27}$$

From Table IV, we see that besides $d_{\mathcal{U}}$ and $\Lambda_{\mathcal{U}}$, the introduced new free parameters are

$$\begin{aligned} &C_L^{db}, \quad C_R^{db}, \quad C_L^{sb}, \quad C_R^{sb}, \\ &C_L^{uu}, \quad C_R^{uu}, \quad C_L^{dd}, \quad C_R^{dd}, \end{aligned} \tag{28}$$

which denote the couplings of unparticle to SM particles. First four parameters are strongly correlated by $B_{d,s} - \bar{B}_{d,s}$ mixing phenomena as shown in Eq.(12). If we regard all these parameters to be real number and set $\Lambda_{\mathcal{U}} = 1$ TeV and $d_{\mathcal{U}} = 1.5$ as we do in the $B_{d,s} - \bar{B}_{d,s}$ analysis, 8 free parameters are involved for $B_{u,d} \to (\pi, K)\pi$ in the unparticle physics.

In order to fit to the data for the $B_{u,d} \to (\pi, K)\pi$ decays with these 8 parameters, we perform the minimum χ^2 analysis. According to current experimental observations, the amount of available data for $B_{u,d} \to (\pi, K)\pi$ decays is 17 as their world averages are displayed in Table V Besides the BRs, the important quantities to display the new physics effects are the direct and mixing induced CPAs, where they could be briefly defined through

$$\mathcal{A}_f \equiv \frac{|\lambda_f|^2 - 1}{1 + |\lambda_f|^2}\,, \quad S_f \equiv \frac{2\mathrm{Im}(\lambda_f)}{1 + |\lambda_f|^2}\,, \tag{29}$$

with $\lambda_f = e^{i\phi_d} \bar{A}_{\bar{f}}/A_f$, respectively. For direct CPA, f could be any possible final states; however, for mixing induced CPA, f could only be CP eigenstates. Since the PQCD prediction for the SM decay amplitudes has sizable error as shown in Table III, it should be considered for the minimum χ^2 analysis. Therefore, we define the χ^2 to be

$$\chi^2 = \sum_{i=1}^{n} \frac{(t_i - e_i)^2}{\sigma_i^{exp\ 2} + \sigma_i^{thr\, 2}}\,. \tag{30}$$

e_i and t_i denote the experimental data for i'th observable and its theoretical prediction within unparticle contribution, respectively. σ_i^{exp} is the experimental error of i'th observable, while

TABLE V: The experimental data for $B_{u,d} \to (\pi,\, K)\pi$ decays and comparison between the experimental data and the theoretical predictions with and without unparticle contribution. The BRs are order of 10^{-6}. The data is updated by September 2007. 'w/o' means 'without unparticle contribution'. χ^2 contributions of each observables are shown.

observables	data	theory (w/o)	theory	χ^2 (w/o)	χ^2
$\mathcal{B}(K^0\pi^+)$	23.1 ± 1.0	23.5 ± 12	23.1 ± 11	0.001	0.0
$\mathcal{B}(K^+\pi^0)$	12.9 ± 0.6	13.0 ± 6.2	12.7 ± 6.0	0.001	0.001
$\mathcal{B}(K^+\pi^-)$	19.4 ± 0.6	19.7 ± 10	20.3 ± 10	0.001	0.007
$\mathcal{B}(K^0\pi^0)$	9.9 ± 0.6	8.8 ± 4.9	9.5 ± 5.1	0.046	0.006
$\mathcal{A}_{CP}(K^0\pi^+)$	0.009 ± 0.025	0.0 ± 0.0	0.0 ± 0.0	0.13	0.13
$\mathcal{A}_{CP}(K^+\pi^0)$	0.050 ± 0.025	-0.017 ± 0.068	0.074 ± 0.068	0.84	0.11
$\mathcal{A}_{CP}(K^+\pi^-)$	-0.097 ± 0.012	-0.099 ± 0.073	-0.11 ± 0.075	0.001	0.03
$\mathcal{A}_{CP}(K^0\pi^0)$	-0.14 ± 0.11	-0.065 ± 0.040	-0.058 ± 0.036	0.41	0.50
$S_{K_S\pi^0}$	0.38 ± 0.19	0.74 ± 0.08	0.74 ± 0.07	3.1	3.2
$\mathcal{B}(\pi^+\pi^0)$	$5.59^{+0.41}_{-0.40}$	4.03 ± 2.53	4.05 ± 2.53	0.37	0.36
$\mathcal{B}(\pi^+\pi^-)$	5.16 ± 0.22	6.80 ± 4.43	7.11 ± 4.43	0.14	0.19
$\mathcal{B}(\pi^0\pi^0)$	1.31 ± 0.21	0.23 ± 0.13	1.33 ± 0.30	19	0.002
$\mathcal{A}_{CP}(\pi^+\pi^0)$	0.06 ± 0.05	0.00 ± 0.01	0.055 ± 0.018	1.6	0.01
$\mathcal{A}_{CP}(\pi^+\pi^-)$	0.38 ± 0.07	0.17 ± 0.10	0.38 ± 0.14	2.8	0.0
$\mathcal{A}_{CP}(\pi^0\pi^0)$	$0.48^{+0.32}_{-0.31}$	0.64 ± 0.23	0.53 ± 0.13	0.17	0.024
$S_{\pi^+\pi^-}$	-0.61 ± 0.08	-0.55 ± 0.44	-0.55 ± 0.42	0.021	0.023

σ_i^{thr} is theoretical error propagated from the errors of topological parameters obtained from PQCD. Since there are 8 free parameters involved in our analysis, the degree of freedom (d.o.f) for the fitting is $(17-8)=9$. As for the angle γ, we use the values of $\gamma = (63^{+15}_{-12})^\circ$ from the PDG 2006 [36]. Consequently, by imposing the mixing constraints of $C^{db}_{L(R)}$ and $C^{sb}_{L(R)}$ displayed in Eq. (12), we find the optimized values of unparticle parameters as follows:

$$C_L^{db} = 3.3\times 10^{-4},\quad C_R^{db} = 4.6\times 10^{-4}\ ,\quad C_L^{sb} = 7.6\times 10^{-4},\quad C_R^{sb} = 11.2\times 10^{-4}\,,$$
$$C_L^{uu} = 5.0,\qquad C_R^{uu} = 12.0\ ,\quad C_L^{dd} = 4.2,\qquad C_R^{dd} = 11.2 \tag{31}$$

with $\chi^2 = 4.6$, compared to $\chi^2 = 28.8$ without the unparticle contributions. From above

TABLE VI: Numerical values of unparticle amplitudes for each decay mode. 'Abs' represents the magnitude of the amplitude in unit of 10^{-5}GeV. 'Arg' represents the CP conserving phase of unparticle in radian unit.

decay mode	Abs	Arg	decay mode	Abs	Arg
$K^0\pi^+$	18.9	-1.6	$\pi^+\pi^0$	0.6	1.6
$K^+\pi^0$	10.8	1.6	$\pi^+\pi^-$	3.5	1.6
$K^+\pi^-$	2.2	-1.6	$\pi^0\pi^0$	9.9	1.6
$K^0\pi^0$	2.9	1.6			

results, we see clearly $B_{d,s} - \bar{B}_{s,d}$ mixings give strict constraints on $C^{db}_{L(R)}$ and $C^{sb}_{L(R)}$ that lead to FCNCs at tree level. We compare the experimental data to the theoretical predictions with and without unparticle contributions in Table V, in detail. And also, the numerical values of unparticle contributions are given in Table VI for comparing with the SM contribution given in Table III. Strikingly, we can see that the large experimental data of BR for $B_d \to \pi^0\pi^0$ can be quite well accommodated with unparticle contributions. Moreover, some anomalous observables of direct CPA of $B^+ \to K^+\pi^0$ and $B_d \to \pi^+\pi^-$ can be explained. However, the puzzle of mixing induced CPA of $B_d \to K_S\pi^0$ could not be resolved well by unparticle contributions. As many authors argued, sizable non-SM weak phase is required in order to fit to the data of $S_{K_S\pi^0}$ [38]. Since the unparticle contributions do not carry any extra weak phase, it turns out to be very hard to fit to the data.

IV. SUMMARY AND CONCLUSIONS

In summary, we have studied the effects of unparticle on $B_{u,d}$ decays with $B_{d,s} - \bar{B}_{d,s}$ mixing constraints. For simplicity, we concentrate on vector unparticle and set the scale of unparticle operator $\Lambda_{\mathcal{U}}$ to be 1 TeV. With the lattice QCD results of JLQCD and (HP+JL)QCD on the non-perturbative quantity of $f_{B_d}\sqrt{\hat{B}_{B_q}}$, we start to calculate the SM predictions for $M^{q,SM}_{12}$. Accordingly, from the current data we extract the available space for the new physics effects in model independent way. When the unparticle effects are included to $\Delta B = 2$ processes, we find that the current experimental data for ΔM_d, ΔM_s and ϕ_d could give strict constraints on unparticle parameters as well as scaling dimension $d_{\mathcal{U}}$. The (HP+JL)QCD case could not give a solution for $d_{\mathcal{U}}$, while all value of $d_{\mathcal{U}}$ is possible

in JLQCD case. However, we see that more accurate SM prediction for the $B_q^0 - \bar{B}_q^0$ mixing amplitude in future would give strong constraint on $d_{\mathcal{U}}$. In order to understand whether the unparticle effects could satisfy all measurements in exclusive $B_{u,d} \to (\pi,\, K)\pi$ decays, we utilize the minimum χ^2 analysis to search for the solutions of free parameters. Interestingly, after we fix $d_{\mathcal{U}} = 1.5$ for the specific unparticle scaling dimension, we find that the unsolved problem of large BR for $B_d \to \pi^0\pi^0$ could be explained excellently in the framework of unparticle physics. Moreover the discrepancy between the standard model estimation and data for the direct CPA of $B^+ \to K^+\pi^0$ and $B_d \to \pi^+\pi^-$ could be reconciled very well. However, the puzzle of the mixing induced CPA of $B_d \to K_S\pi^0$ could not be resolved well in unparticle physics.

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